

AD-A122 863

CONFERENCE ON ORDINARY AND PARTIAL DIFFERENTIAL
EQUATIONS 29 MARCH TO 2 APRIL 1982(U) DUNDEE UNIV
(SCOTLAND) 02 APR 82

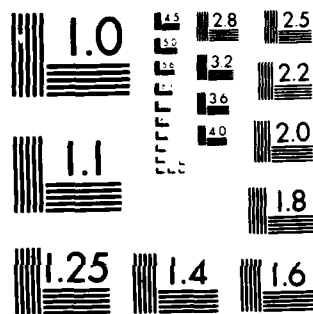
1//

UNCLASSIFIED

F/G 12/1

NL

END
DATE
FILMED
GPO
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

2

UNIVERSITY OF DUNDEE

CONFERENCE ON ORDINARY AND PARTIAL
DIFFERENTIAL EQUATIONS

29 March to 2 April 1982

DTIC
ELECTE
DEC 28 1983
H

ABSTRACTS OF LECTURES

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

92 12 97 121

AD A122863

FILE COPY

A U Afuwape

On Lyapunov-Razumikhin approach to certain
third-order equations with delay

In this paper, we consider the asymptotic stability for equations of the form:

$$(*) \quad x'''(t) + ax''(t) + bx'(t) + h(x(t), x(t-\tau)) = 0$$

via the Lyapunov-Razumikhin approach. For the case when $\tau = 0$, a Lyapunov function is constructed and conditions which reduce to the generalized Routh-Hurwitz criteria for uniform asymptotic stability are obtained. The constructed Lyapunov function is also converted to a Lyapunov functional. This functional is used to give necessary conditions on a , b and h for which equation (*) is asymptotically stable.

N D Alikakos

Invariance in the mean in reaction-diffusion
equations and applications

The system

$$u_t = D\Delta u + B(u) \quad \left. \frac{\partial u}{\partial n} \right|_{\partial\Omega} = 0, \quad \Omega \subseteq \mathbb{R}^n \quad (*)$$

$u = \text{col}(u_1, \dots, u_N)$, $B(u) = \text{col}(u_1, \dots, u_N)$, $D =$ positive definite symmetric $N \times N$ matrix, is naturally associated to the system of ordinary differential equations

$$u_t = B(u). \quad (**)$$

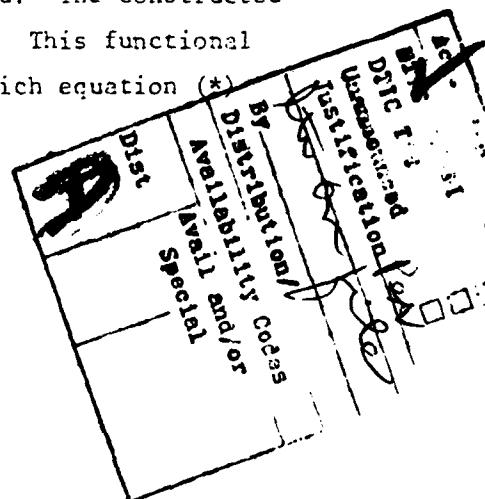
Often there exists a set in \mathbb{R}^n that is positively invariant for (**) and the question is if this set enjoys similar invariance properties with respect to (*). We restrict our attention to sets

$$M^c = \{z | G(z) \leq c\}, \quad G: \mathbb{R}^N \rightarrow \mathbb{R}^+, \quad C^2.$$

It is known that in general the set M^c is not invariant for (*). This is in the nature of things and it is a manifestation of strong coupling. Motivated by this we introduce a new class of sets given by

$$M^c = \left\{ \xi(x) \mid \int_{\Omega} G(\xi(x)) dx \leq c \right\} \quad (m(\Omega) = 1).$$

First we characterize the sets M^c that are positively invariant for (*). The essence of this characterization (which is variational in character) is that



the symmetrization of the matrix $D \left(\frac{\partial^2 G(z)}{\partial z_i \partial z_j} \right)$ has to be positive semidefinite

for all z . This is a quantitative condition that relates the geometry of the set and the diffusion matrix and in particular implies that G is convex. We recapture the conventional invariance of M^c by considering the sets

$$M_p^c = \{ \xi(x) / \left(\int_{\Omega} G^p(\xi(x)) dx \right)^{1/p} \leq c \}$$

and letting $p \rightarrow \infty$. We characterize the sets M_p^c that are invariant for all p and some c . As an application we discuss the asymptotic behaviour of the solutions of

$$u_t = D \Delta u + [1 - \langle Au, u \rangle] u, \quad A = \text{positive definite symmetric.}$$

Finally we mention briefly some aspects of the corresponding theory of invariance when in addition to (*) a set of linear conservation laws has to be satisfied.

A H Azzam

Boundary value problems for elliptic and
parabolic equations in domains with corners

The paper concerns initial - Dirichlet and initial - mixed boundary value problems for parabolic equations.

$$a_{ij}(x,t)u_{x_i x_j} + a_i(x,t)u_{x_i} + a(x,t)u - u_t = f(x,t),$$

$x = x_1, \dots, x_n$, $n \geq 2$. We consider the case of nonsmooth boundaries and discuss the smoothness of the solution up to the boundary.

The dependence of this smoothness upon the values of the "angles" on the boundary is given explicitly. The case of the elliptic boundary value problem is also investigated.

P W Bates

Bifurcation and stability of periodic travelling waves
for a reaction-diffusion system

Consider the system

$$(1) \quad \begin{cases} u_t = u_{xx} + k(1 - \rho)u - \phi_1(\beta)v \\ v_t = v_{xx} + k(1 - \rho)v + \phi_1(\beta)u, \quad -\pi \leq x \leq \pi, \quad t \geq 0, \end{cases}$$

where ρ and β are given by $u = \rho \cos \beta$, $v = \rho \sin \beta$, ϕ_1 is smooth and 2π -periodic and k is a parameter.

Theorem For each nonzero integer j , if $k > j^2$ and $k \neq 5j^2 - n^2$ for $n = 1, 1, \dots, 2|j| - 1$, then for ϕ_1 sufficiently small (1) has a 2π -periodic travelling wave solution which, in the $u - v$ plane, loops around the origin j times. Furthermore, if $k > 5j^2 - 1$ then this solution is stable and if $k < 5j^2 - 1$ it is linearly unstable.

H Behncke

The Dirac equation with an anomalous magnetic moment II

Though it is well known, that the electron possesses an anomalous magnetic moment, this term has not been considered so far in the mathematical investigations of the Dirac equation [3,4,5,6], and it has at most been treated by perturbation methods in the physical literature. Recent investigations of Barut [1] and this author [2] make it desirable to study the interaction due to the anomalous magnetic moment non-perturbatively, because this term is rather singular. This singularity leads to a number of interesting results. Most notable among these is the essential selfadjointness of the Hamiltonian for almost all potentials and the small distance behaviour of the wave functions. The essential spectrum of the Dirac Hamiltonian is the main object of this paper. The investigations are carried out essentially only for spherically symmetric potentials.

V Benci, A Capozzi and D Fortunato

Periodic solutions of a class of Hamiltonian systems

Let $H \in C^1(\mathbb{R}^{2n}, \mathbb{R})$ and consider the Hamiltonian system of $2n$ ordinary differential equations

$$(1) \quad \dot{p} = -H_q(p, q), \quad \dot{q} = H_p(p, q)$$

where p and q are n -tuples and \cdot denotes $\frac{d}{dt}$. If H grows more than quadratically (in a suitable way) in both the variables p and q , Rabinowitz (cf. Comm. Pure Appl. Math. 31 (1978), 157-184) has proved that for any $T > 0$ (1) possesses a nonconstant T -periodic solution.

Here we consider Hamiltonian functions of the type

$$H(p, q) = \sum_{ij} a_{ij}(q) p_i p_j + V(q)$$

where $[a_{ij}(q)]$ is a positive-definite matrix and $V(q)$ grows more than quadratically at infinity.

Under suitable assumptions on the growth of $a_{ij}(q)$, we prove that for any

Distribution of the zeros of Bôcher's pairs with respect to
second order hom. linear differential equations

As a generalization of certain former results ([1]-[2]) some lower estimates can be obtained concerning the distance of the zeros of the pairs

(1) $\phi = \phi_1 y - \phi_2 \frac{y'}{\sigma}$, $\Psi = \psi_1 y - \psi_2 \frac{y'}{\sigma}$, $\sigma = \sqrt{p}$ where y is a non-trivial solution of the equation (2) $y'' + p(x)y = 0$, $x \in I \subset \mathbb{R}$, $p \in C(I)$, $p > 0$.

Theorem Let us denote the zeros - if any - of ϕ by x_i ($i=0,1,2,\dots$) and those of Ψ by x'_i ($i=1,2,\dots$) where $0 = x_0 < x'_1 < x_1 < x'_2 < \dots$ and define $\alpha(x)$ by $\alpha(x) = \arctan \frac{\phi}{\Psi}$ $\alpha(0) = 0$. Assume furthermore

(3) $\phi_i, \psi_i \in C_1(I)$, $\{\phi_1, \phi_2\} \neq 0$, $\{\psi_1, \psi_2\} \neq 0$, $\{\phi, \Psi\} \neq 0$

(4) $D = \phi_1 \psi_2 - \phi_2 \psi_1 < 0$, $D = \{\phi, \Psi\}^2 - 4\{\phi_1, \phi_2\} \cdot \{\psi_1, \psi_2\} < 0$, $x \in I$

where $\{\phi_1, \phi_2\} = \frac{1}{\sigma}(\phi_1' \phi_2 - \phi_2' \phi_1) + \frac{\sigma'}{2} \phi_1 \phi_2 + \phi_1^2 + \phi_2^2$, $\{\phi, \Psi\} = \frac{1}{\sigma}(\phi_2' \psi_1 - \phi_1' \psi_2 + \psi_1' \phi_2 - \psi_2' \phi_1) + \frac{\sigma'}{2} (\phi_1 \psi_2 + \phi_2 \psi_1) + 2(\phi_1 \psi_1 + \phi_2 \psi_2)$

(5) $f(v(u)) < f(v(u - \frac{\pi}{2}))$, $i\pi \leq u \leq (i + \frac{1}{2})\pi$, ($i=1,2,\dots$)

where $f(x) = -\frac{2}{\{\phi, \Psi\}} [\{\psi_1, \psi_2\} \sin^2 \alpha + \{\phi_1, \phi_2\} \cos^2 \alpha]$

and $x=u(v)$ is the inverse of the monotonic function $u=\alpha(x)$, then

(6) $\tau(x_i) > i\pi - \frac{\pi}{2}$, $i = 0,1,2,\dots$

(6) $\tau(x'_i) > (i - \frac{1}{2})\pi - \frac{\pi}{2}$, $i = 1,2,\dots$

where

(7) $\tau(x) = -\int_0^x \frac{\sigma(u)}{D(u)} [\{\psi_1, \psi_2\}_u \sin^2 \alpha(u) + \{\phi_1, \phi_2\}_u \cos^2 \alpha(u)] du$.

By an analysis it will be shown that the above conditions can be satisfied.

If $\phi_1=1$, $\phi_2=0$, $\psi_1=0$, $\psi_2=-\sigma$, then $\tau(x)$ reduces to $\int_0^x \sigma(u) du$. (See [1]-[2].)

[1] Á Elbert: On the zeros of solutions of ordinary second order differential equations. Publ. Math. Debrecen 15 (1968) 13-17.

[2] I Bihari: Notes on the eigenvalues and zeros of the solutions of half-linear second order ordinary differential equations. Periodica Math. Hung. 7(2) (1976) 117-125.

T R BlowsLimit cycles of polynomial differential equations

The study of limit cycles of two-dimensional autonomous differential equations of the form

$$dx/dt = P_n(X,y) , \quad dy/dt = Q_n(X,y) ,$$

where P_n and Q_n are polynomials of degree at most n , is a well known and long-standing question.

Results are given for the cases $n = 2, 3$ and 5 which have been obtained using successive bifurcation of limit cycles from a critical point. These so called fine focus techniques require for their implementation the calculation of certain large polynomials in several variables; these calculations have been performed with the aid of a computer.

For example, in the case $n = 3$, a family of systems which have five small amplitude limit cycles is given.

K J BrownMultiple steady state solutions for nonlinear systems of elliptic equations (Describing Interacting Populations.)

Let Ω be a bounded region in \mathbb{R}^n . Solutions of the nonlinear system of elliptic equations on Ω

$$-d_1 \Delta u = f(u,v)u$$

$$-d_2 \Delta v = g(u,v)v$$

correspond to steady states in which two interacting species u and v can co-exist on Ω . It is assumed that on $\partial\Omega \equiv 0$ and $v(x) = \phi(x) \geq 0$. It is shown that, if f and g satisfy hypotheses consistent with both populations being self-limiting and the interaction being of predator-prey, competition or cooperation type, then the equations can be decoupled and the existence of multiple solutions investigated.

R C BrownPerturbation invariance of the Dirichlet index under minimal conditions

A new interpretation of the Dirichlet index for symmetric differential operators with positive coefficients under minimal conditions is developed. In particular it is shown that the index is minimal if and only if a certain associated vector valued operator is "limit-point" and also that the index is invariant under positive t -bounded perturbations. The results complement recent discoveries of

P J Browne

Multiparameter Problems: The Last Decade

L Brüll

A new abstract existence theory for nonlinear
Schrödinger and wave equations

In analogy to the method of Peano for ordinary differential equations we construct for the initial value problem of the nonlinear Schrödinger and wave equation in a Hilbert space solutions by using their integral representation. In this procedure we can do without Lipschitz-estimates for the nonlinear part. With a suitable notion of the integral in a Hilbert space and with a theorem of Ascoli-type for weak convergence we get local weak solutions, which could be continued to a global weak solution, if an a-priori-estimate is available. With this the existence of strong, global solutions can be proved easily.

Similar to the theorem of Peano we couldn't guarantee the uniqueness of the solutions.

With the gained theorems results of L A Medeiros (On a New Class of Nonlinear Wave Equations, J. Math. Anal. Appl., 69, 252-262 (1979)) and M Reed (Abstract Nonlinear Wave Equations, Springer Lecture Notes in Mathematics, No 507, Heidelberg (1976)) can be generalized.

J A Burns

Approximation of nonlinear neutral functional differential equations

It has been known for some time that a class of initial-boundary value problems for hyperbolic partial equations can be transformed to equivalent initial value problems for neutral functional differential equations. Neutral equations also serve as models for certain problems in elasticity.

In this paper we motivate the study of neutral equations by briefly discussing physical problems from elasticity and lossless transmission lines that are modeled by such equations. These equations are formulated as Cauchy problems in an appropriate Hilbert space, and approximation results from semigroup theory are employed to develop approximation schemes. Convergence results and a discussion of resulting numerical algorithms are given. Spline based approximations are used to illustrate the theory.

L CattabrigaResults and problems on the Gevrey surjectivity of
linear differential operators

In this lecture a sufficient condition for the existence of a solution u in a Gevrey space $\Gamma^d(\mathbb{R}^n)$, d a rational number ≥ 1 , $n \geq 2$, to a linear partial differential equation with constant coefficients $P(D)u = f$ is proved when $f \in \Gamma^d(\mathbb{R}^n)$. Here $D = (D_1, \dots, D_n)$, $D_j = -i\partial/\partial x_j$, and $\Gamma^d(\mathbb{R}^n)$ is the space of all C^∞ complex valued functions f in \mathbb{R}^n such that for every compact set $K \subset \mathbb{R}^n$ there exists a constant $c(K)$, dependent on f , such that

$$\sup_{x \in K} |D^\alpha f(x)| \leq c(K) |\alpha|^{d+1} \Gamma(d|\alpha| + 1) \text{ for every multi-index } \alpha \text{ of non-negative integers.}$$

Some open problems are discussed.

P A ClarksonA connection formula for the second Painlevé transcendent

We consider a particular case of the Second Painlevé Transcendent

$$y'' = xy + 2y^3.$$

It is known that if $y(x) \sim k \operatorname{Ai}(x)$ as $x \rightarrow \infty$ then if $0 < k < 1$

$$y(x) \sim \frac{d}{|x|^{1/4}} \sin\left\{\frac{2}{3}|x|^{3/2} - \frac{3}{4}d^2 \ln|x| - c\right\} \text{ as } x \rightarrow -\infty$$

where $d(k)$ and $c(k)$ are the connection formulae for this nonlinear ordinary differential equation.

The lecture shows that

$$d^2(k) = -\pi^{-1} \ln(1-k^2)$$

which confirms the numerical estimates of Ablowitz and Segur.

L CollatzInclusion theorems for singular and free boundary value problemsD L ColtonThe inverse scattering problem for
acoustic and electromagnetic waves

The inverse scattering problem under consideration is to determine the physical properties of an obstacle situated in a homogeneous medium from a knowledge of its effect on an incoming time harmonic plane wave. In particular it is desired to determine either the shape or surface impedance of the obstacle

from a knowledge of the far field pattern of the scattered wave. The basic difficulty in trying to solve such problems lies in the fact that they are both nonlinear and improperly posed. In this lecture we shall examine the following topics: (1) The class of physically realizable far field patterns considered as a subset of $L^2(\Omega)$ where Ω is the unit sphere, (2) The uniqueness of solutions to the inverse scattering problem, and (3) The stabilization of the inverse scattering problem and its reformulation as a problem in constrained optimization. Problems that are in need of further investigation will be discussed.

J Donig

Positive eigensolutions and the lower spectrum of
Schrödinger operators

Let Ω be a domain in \mathbb{R}^m ($m \geq 2$), let q be a real-valued measurable function on Ω , and let

$$\tau := -\Delta + q.$$

Furthermore, suppose that q satisfies a local Stummel condition and is such that the minimal Schrödinger operator

$$T_0 := \tau \quad C_0^\infty(\Omega)$$

in $L^2(\Omega)$ is bounded from below.

We prove, for appropriate values of λ , that the Schrödinger equation

$$\tau u = \lambda u$$

has a weak solution which is positive almost everywhere. In addition, we give a criterion when an m -sectorial extension T of T_0 has a finite lower spectrum.

N X Dũng

Essential self-adjointness and self-adjointness for
generalized Schrödinger operators

Consider a formally self-adjoint operator of the form

$$T = \sum_{|\alpha|, |\beta| \leq m} (-1)^{|\alpha|} D^\alpha a_{\alpha\beta}(x) D^\beta + q(x) \quad \text{on } L^2(\mathbb{R}^n).$$

General assumptions

1. Uniform strong ellipticity for T ,
2. $a_{\alpha\beta} \in C^{\max(|\alpha|, |\beta|)}(\mathbb{R}^n)$,
3. The conditions for Gårding's inequality hold for $P = T - q$.

.../continued

Theorem 1 If q is measurable and locally bounded, and $q(x) \geq -q^*(|x|)$, where $q^*(r)$ is monotone non-decreasing in $r > 0$, and $q^*(r) = 0$ ($r^{2m/(2m-1)}$) as $r \rightarrow +\infty$, then T is essentially self-adjoint on $C_0^\infty(\mathbb{R}^n)$.

Theorem 2 Suppose $a_{\alpha\beta}$, with $|\alpha| + |\beta| \leq 2m$, has bounded derivatives up to order m , then there exists a positive constant C , depending on n, m , the ellipticity constant, the moduli of continuity of $a_{\alpha\beta}$ with $|\alpha| = |\beta| = m$, $\|D^\gamma a_{\alpha\beta}\|_{L^\infty}$ with $|\alpha| + |\beta| \leq 2m$ and $|\gamma| \leq \max(|\alpha|, |\beta|)$, with the following property:

If $q \geq 1$ and for $0 < |\alpha| \leq m$, $|D^\alpha q(x)| \leq C q(x)^{\frac{1+|\alpha|}{2m}}$, then T is self-adjoint with domain $D(T) = H^{2m}(\mathbb{R}^n) \cap D(q)$.

M S P Eastham

Asymptotic analysis for a critical class of
fourth-order differential equations

The asymptotic theory of the equation

$$(x^\alpha y'')'' - b(x^\beta y')' + cx^\gamma y = 0 \quad (x \rightarrow \infty)$$

presents difficulties when $\gamma = \beta - 2$. A partial analysis of this case has been made using a number of ad hoc methods. When $\alpha = 0$, the most complete results are those of Paris and Wood, who solved the equation in terms of generalized hypergeometric functions. In this lecture, the powers of x are replaced by general coefficients p, q, r and a general asymptotic theory is developed. In this theory, the case where $(qr)'/(qr) \sim (\text{const.})(r/q)^{1/2}$ corresponds to $\gamma = \beta - 2$ and is shown to occupy a critical position. The asymptotic results which are obtained for this critical case substantially cover the previous ones derived from the ad hoc methods for the powers of x coefficients.

D E Edmunds

Entropy numbers, s-numbers and eigenvalues

The lecture will survey recent developments in the theory of bounded, not necessarily compact, linear maps acting in a Banach space, emphasis being placed upon the connections between geometric quantities, such as the entropy numbers of such maps, and more analytical objects, such as eigenvalues. Connections with eigenvalue problems for elliptic operators will be stressed.

A ElbertOscillation and nonoscillation theorems for a
half-linear second order differential equation

We consider the ordinary second order differential equation of the form

$$(*) \quad ((x')^{\frac{*}{n}})' + nq(t)x^{\frac{*}{n}} = 0$$

where the number n is positive and real, the power function $u^{\frac{*}{n}}$ denotes the function $|u|^n \cdot \text{sign } u$, and $q(t)$ is continuous on the interval $[t_0, \infty)$. The classification of this equation into oscillatory and nonoscillatory classes is possible. We give several criteria for deciding this classification. As examples we show some in the simplest situation.

The relation

$$\lim_{T \rightarrow \infty} \int_{t_0}^T q(t) dt = \infty$$

implies the oscillation of the solution of (*).

If the function

$$H(T) = \frac{\int_{t_0}^T \left\{ \int_{t_0}^t q(\tau) d\tau \right\} dt}{T}$$

is bounded, and $\limsup_{T \rightarrow \infty} H(T) > \liminf_{T \rightarrow \infty} H(T)$, then the equation (*) is oscillatory.

If this function $H(T)$ has a finite limit C and

$$\limsup_{T \rightarrow \infty} \frac{\int_{t_0}^T \left| C - \int_{t_0}^t q(\tau) d\tau \right|^{\frac{n+1}{n}} dt}{T} > 0$$

then (*) is oscillatory.

W D EvansOn the distribution of eigenvalues of Schrödinger operators

Let Ω be an unbounded domain in R^n , $n \geq 1$ and let F be a tessellation of R^n by closed cubes Q with disjoint interiors (but not necessarily congruent). Let q be a real-valued function in $L_{loc}^s(\Omega)$ for some s in $[n/2, \infty]$ if $n \geq 3$, in $(1, \infty]$ if $n = 2$ and in $[1, \infty]$ if $n = 1$ and suppose that with the notation $q_Q = |Q|^{-1} \int_Q q$ and $\rho_Q = \gamma_s |Q|^{-1/s} \|q - q_Q\|_{L^s(Q)}$ (where γ_s is the norm of the embedding of $W^{1,2}((0,1)^n)$ in $L^{2s/s-1}((0,1)^n)$ and $|Q|$ is the volume of Q) we have

$$\sup \{ |Q|^{-1/s} \|q_Q\|_{L^s(Q)} \} < \infty, \quad \theta := \inf \{ q_Q - \rho_Q \} > -\infty$$

and $\delta := \sup_{Q \in F} [|Q|^{2/n} \rho_Q] < 1$. Then the Dirichlet operator T defined by the form of $-\Delta + q$ in $L^2(\Omega)$ is a self-adjoint operator whose spectrum lies in $[\theta, \infty)$ and essential spectrum in $[\theta_e, \infty)$, where

$$\theta_e = \liminf_{|x_Q| \rightarrow \infty} [q_Q - \rho_Q],$$

x_Q being the centre of Q . For $\lambda < \theta_e$ we have that $N(\lambda, \Omega)$, the number of eigenvalues of T less than λ , satisfies

$$|N(\lambda, \Omega) - \omega_n (2\pi)^{-n} \Lambda| \leq \frac{1}{2} n \delta \omega_n (2\pi)^{-n} + O(\Lambda^{(n-1)/n} N_0^{1/n} + N_0)$$

where ω_n is the volume of the unit ball in \mathbb{R}^n , $\Lambda = \sum_{q_Q < \lambda} |Q| (\lambda - q_Q)^{n/2}$ and N_0

is the number of cubes Q in F for which $q_Q - \rho_Q \leq \lambda$. This general result can be specialised to give asymptotic results in the problem when $q(x) \rightarrow \infty$ (when T has a wholly discrete spectrum) and $\lambda \rightarrow \infty$ and also in the problem when $q(x) \rightarrow 0$ (when $[0, \infty)$ lies in the essential spectrum of T) and $\lambda \rightarrow 0^-$. Moreover, results concerning $N(0, \mathbb{R}^n)$ when $q \in L^{n/2}(\mathbb{R}^n)$, $n \geq 3$, can be obtained. Analogous results hold when T is the Neumann operator defined by the form of $-\Delta + q$ in $L^2(\Omega)$.

W N Everitt

Two examples of the Hardy-Littlewood type of integral inequalities

This lecture reports on joint work with W D Evans and W K Hayman.

The integral inequalities are of the type

$$\left(\int_a^b \{p|f'|^2 + q|f|^2\} dx \right)^2 \leq K \int_a^b w|f|^2 dx \int_a^b w|w^{-1}(-(pf)'+qf)|^2 dx$$

where $p, q, w : [a, b) \rightarrow \mathbb{R}$, $p^{-1}, q, w \in L_{loc}^1[a, b)$, $w(x) > 0$ for almost all $x \in [a, b)$, and $f : [a, b) \rightarrow \mathbb{C}$ is chosen so that the integrals on the right-hand side are convergent. If p, q, w are suitably chosen the integral on the left-hand is finite but may only be conditionally convergent; hence the notation $\rightarrow b$.

The following examples, which are important in the general theory of the inequality are considered:

$$(i) \quad a = 0 \quad b = \infty \quad p(x) = w(x) = 1 \quad q(x) = -x \quad (x \in [0, \infty))$$

$$\left(\int_0^\infty \{|f'(x)|^2 - x|f(x)|^2\} dx \right)^2 \leq K \int_0^\infty |f(x)|^2 dx \int_0^\infty |f''(x) + x f(x)|^2 dx$$

$$(ii) \quad a = 0 \quad b = \infty \quad p(x) = w(x) = 1 \quad q(x) = x^2 + 1 \quad (x \in [0, \infty))$$

$$\left(\int_0^\infty \{|f'(x)|^2 + (x^2 + 1)|f(x)|^2\} dx \right)^2 \leq K \int_0^\infty |f(x)|^2 dx \int_0^\infty |-f''(x) + (x^2 + 1)f(x)|^2 dx.$$

P C Fife

Laminar flame theory with multiple reactions

The problem of determining the structure and velocity of plane flames is formulated. The standard high-activation-energy approach for a single one-step reaction is reviewed. Multiple-step reactions are then discussed under the assumption that the various activation energies are either large or zero. The possibility of many flame types for a given reaction mechanism is brought out, as is the effect of cross diffusion of the reacting species.

J Fleckinger

Singular numbers of operators of Schrödinger type
with complex potentials

We obtain an asymptotic estimate for the singular numbers of non self adjoint operators of Schrödinger type: for example we obtain an estimate for the eigenvalues of H^*H when $H = -\Delta + q$, with q complex valued function, is defined on Ω unbounded domain in \mathbb{R}^n .

C T Fulton

Some limit circle eigenvalue problems and asymptotic
formulae for eigenvalues

This lecture will report on recent work with F V Atkinson on asymptotics of Sturm-Liouville eigenvalues for singular problems involving limit circle endpoints. Asymptotic formulae for eigenvalues and eigenfunctions of $-y'' + qy = \lambda y$ are well known for problems having regular endpoints, but considerably less is known when one endpoint is singular.

For the case of the half line $[0, \infty)$ with the L.C. case at ∞ , asymptotic formulae for the positive and negative eigenvalues, showing their dependence on the parameter indexing the boundary condition at ∞ , are obtained under assumptions on q which are general enough to include the cases:

$$(1) \quad q_1(x) = -x^\alpha, \quad 2 < \alpha < \infty,$$

and

$$(2) \quad q_2(x) = -e^{ax}, \quad a > 0.$$

For the case of the finite interval $(0, b]$, asymptotic formulae for the positive eigenvalues are obtained when $x = 0$ is nonoscillatory under assumptions on q which are general enough to include the nonintegrable case

$$(3) \quad q_3(x) = -\frac{C}{x^\alpha}, \quad 1 \leq \alpha < 2, \quad C \in (-\infty, \infty).$$

/continued

Also, for the case of $(0, b]$, asymptotics for the positive and negative eigenvalues are obtained when $x = 0$ is oscillatory under assumptions on q which are general enough to include the strongly nonintegrable case

$$(4) \quad q_4(x) = -\frac{1}{x^\alpha}, \quad 2 < \alpha < \infty.$$

The case $\alpha = 2$ in (3) splits into three cases depending on whether $C \in (-\infty, -1/4)$, $C \in [-1/4, 3/4)$, or $C \in [3/4, \infty)$, and asymptotics for eigenvalues are available from Bessel functions.

For the first problem, these results improve results of A G Alenitsyn, Differential Equations 12, No 2, 298-305 (1977) (Differentsial'nye Uravneniya 12, No 3, 428-437 (1976)). Previous work, obtaining density results for the positive and negative spectrum, has been done by P Heywood, Proc. London Math. Soc. (3) 4, 456-470 (1954), and V P Belogrud and A G Kostyuchenko, Usp. Mat. Nauk, 28, No 2, 227-228 (1973).

I M Gali

Optimal control of systems governed by elliptic operators of infinite order

In the present paper, using the theory of J L Lions we find the set of inequalities defining an optimal control of systems governed by elliptic operator of infinite order. The questions treated in this paper are related to a previous result by I M Gali, et al., but in different direction, by taking the case of operators of infinite order with finite dimension.

M J Goldstein and R L Sternberg*

On a new numerical method for a new class of nonlinear partial differential equations arising in nonspherical geometrical optics

The surfaces $S : z = z(x, y)$ and $S' : z' = z'(x', y')$ of certain classes of optical, radar, and sonar lens antennas satisfy equations of the form

$$\begin{array}{ll} S: & \frac{\partial z}{\partial x} = F, \quad \frac{\partial(z', y')}{\partial(x, y)} = F' \frac{\partial(x', y')}{\partial(x, y)}, \\ & \frac{\partial z}{\partial y} = G, \quad S: \quad \frac{\partial(x', z')}{\partial(x, y)} = G' \frac{\partial(x', y')}{\partial(x, y)}, \end{array} \quad (1)$$

symmetry conditions of the form

$$\begin{array}{ll} z(-x, y) = z(x, y), & z'(-x', y') = z'(x', y'), \\ z(x, -y) = z(x, y), & z'(x', -y') = z'(x', y'), \end{array} \quad (2)$$

and boundary conditions of the form

$$z(x, y) = 0 \quad \text{and} \quad z'(x, y) = 0, \quad (3)$$

on the ellipse: $\Gamma: (x^2/b_0^2 \cos^2 \psi_0) + (y^2/b_0^2) = 1.$

A new numerical method of solving the problem consisting of (1), (2), and (3) is presented and the results are compared with known earlier solutions. Applications to radar and sonar are briefly noted with appropriate references.

R Grimmer

Weak solutions of integrodifferential equations and applications

The integrodifferential equation

$$x'(t) = Ax(t) + \int_0^t B(t-s)x(s)ds + f(t)$$

$$x(0) = x_0 \in D(A)$$

is studied in a Banach space X^- . It is assumed that A and $B(t)$ are closed operators with dense domain with the domain of $B(t)$ containing the domain of A . The concept of weak solution is defined and it is shown that under mild assumptions on $B(t)$, there is a weak solution for every $x_0 \in X^-$ if and only if there is a "resolvent operator" $R(t)$. As an application it is shown that

$$\int_0^t R(t-s)f(s)ds$$

is a solution if and only if it is differentiable.

D B Hinton

Titchmarsh's λ -dependent boundary conditions for Hamiltonian systems

A linear Hamiltonian system $J_y^+ = [\lambda A(x) + B(x)]_y^+$ is considered on an open interval (a,b) where both a and b are singular points. The coefficients $A(x)$ and $B(x)$ are assumed to be locally Lebesgue integrable with $A(x) \geq 0$ and $B(x) = B(x)^*$; $J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$ where I is the $n \times n$ identity matrix. By considering a singular Green's function defined as a limit of regular Green's functions, it is proved that solutions of the differential system defined by the Green's function satisfy Titchmarsh's λ -dependent boundary conditions at the singular points. A formula linking the Titchmarsh-Weyl matrix m -coefficient to certain square integrable solutions is established. These results are developed for the cases where the number of square integrable solutions of the Hamiltonian system is either maximal or minimal.

F A Howes

Exterior boundary value problems for
perturbed equations of elliptic type

The existence and the asymptotic behaviour of solutions of boundary value problems for the elliptic equation $\varepsilon \nabla^2 u = A(x, u) \cdot \nabla u + h(x, u)$ as $\varepsilon \rightarrow 0$ are considered in unbounded regions in \mathbb{R}^N . It is shown that by working within a class of functions that have a restricted growth at infinity, the problem can be studied effectively with the aid of comparison arguments and some recent results of the speaker on related problems in bounded regions. The theory presented here extends the linear theory of Mauss to nonlinear operators, and it clarifies some of the questions he raised concerning the existence and the location of free boundary layers.

R I Jewett

Eigenfunctions and power functions

There is a class of special functions associated with each differential operator of the form $L = \alpha D^2 + \beta D$, where α and β are real-valued analytic functions on an open interval I , α is zero at precisely one point, and β/α' is positive at that point.

The special functions determined by D on the real line include the eigenfunctions, $f_c(x) = e^{cx}$, and the power functions, $\pi_n(x) = x^n$. There are analogous functions determined by the operator L on the interval I . The eigenfunctions and the corresponding expansions are well-known. We shall discuss the power functions of L and the corresponding expansions. These expansions have the form of Taylor series.

H Kalf

Virial theorems in quantum mechanics

For a large class of potentials $q : \mathbb{R}^n \rightarrow \mathbb{R}$ it is well known that an eigenfunction u of the Schrödinger operator $-\Delta + q$ (suitably defined in $L^2(\mathbb{R}^n)$) satisfies the so-called virial theorem

$$(1) \quad 2(u, -\Delta u) = (u, x \cdot \nabla q u) .$$

Adding $2(u, q u)$ on both sides yields

$$(2) \quad 2\lambda \|u\|^2 = (u, (x \cdot \nabla q + 2q)u)$$

where λ is the corresponding eigenvalue. We prove (2) for a class of strongly singular potentials q where the right-hand side of (2) is now to be interpreted merely as a short-hand for

$$(u, (x \cdot \nabla q + 2q)u)$$

and not as a scalar product between functions $L^2(\mathbb{R}^n)$. Since there may be cancellations between the singularities of $x \nabla q$ and those of $2q$, (2) is no longer equivalent to (1).

The consequences of (1) or (2) for the spectrum of $-\Delta + q$ are discussed. It is then shown that a "localized" version of relationship (2) provides a unifying basis for the three main approaches to proving absence of L^2 -solutions of the Schrödinger equation which can be associated with the names of Kato, Agmon and Eidus and Roze.

H G Kaper and A Zettl

Linear transport theory and an indefinite Sturm-Liouville problem

Linear transport processes occur whenever particles move in a host medium, carrying mass, momentum, and energy from one point of the medium to another. Mathematical models of such transport processes involve two operators, one accounting for free streaming of the particles, the other for interactions between the particles and the atoms or molecules of the surrounding host medium. We investigate a time-independent electron transport problem, where the free streaming operator is the multiplicative coordinate operator in $L^2(-1,1)$ and the interaction operator is the Legendre differential operator.

R M Kauffman

On non-normalizable eigenfunction expansions for
ordinary differential operators

The Gelfand-Kostyuchenko theory of generalized eigenfunction expansions deals with the representation $\phi = \int_{-\infty}^{\infty} c(\lambda) f_{\lambda} d(p_{\lambda} f, f)$, where $c(\lambda) = \bar{f}_{\lambda}(\phi)$, where f is a cyclic vector for a subspace containing ϕ , where p_{λ} is the spectral measure for a self-adjoint operator H , and where f_{λ} is a generalized eigenfunction of H for each λ . If H is a self-adjoint operator in $L_2(R)$ associated with an ordinary differential operator L , then f_{λ} is a function and $Lf_{\lambda} = \lambda f_{\lambda}$. While f_{λ} need not be square-integrable, the Gelfand-Kostyuchenko theory nevertheless gives some information about the growth of f_{λ} at infinity.

Although certain portions of the proof of this representation as given in Gelfand-Shilov, Vol. 3, are difficult to follow, a different argument may be given to establish the representation. In this talk, we specialize to the case where H is an ordinary differential operator, state the representation theorem carefully, and apply it to give some new results about self-adjoint ordinary differential operators in $L_2(-\infty, \infty)$ and their associated unitary groups.

M Kisielewicz

Existence of solutions of functional generalized
equations of neutral type

Existence of solutions of functional-differential equations of the form

$$(1) \quad \dot{x}(t) = F(t, x(\cdot), \dot{x}(\cdot)) ,$$

where F is multivalued mapping taking its values in the space of all nonempty compact subsets of R^n , was investigated in the author's paper (Journ. Math. Anal. Appl., 1980) by the assumption that F is strongly Lipschitzian with respect to its third variable. It is the aim of this lecture to present the existence theorem for (1) by the assumption that F satisfies the Caratheodory conditions and is Lipschitzian with respect to its third variable.

R E Kleinman

Recent developments in modified Green's functions

Boundary integral equation formulations of exterior problems for the Helmholtz equation derived either by Green's theorem or a layer ansatz are known to be ill posed at certain characteristic or irregular values of wave number. By modifying the Green's function it was shown by Jones and Ursell that these irregular values may be suppressed. Subsequently it was shown how to modify the Green's function so as to best approximate the actual Green's function for the particular problem in the least squares sense for both Dirichlet and Neumann boundary conditions. In the present paper we show how to modify the Green's function so as to minimize the norm of the modified integral operator and also discuss that modification which minimizes the condition number of the modified operator. The relationship between these various modifications is explored. In addition applications of these methods to the Robin problem is described.

H W Knobloch

Some recent aspects and developments in control theory

The increased efficiency of computer hardware is an incentive for the systems engineer to consider problems of a more and more complex mathematical nature. The lecture will deal with some of these problems which can be formulated by employing state space models, i.e. by describing the system in terms of (linear) ordinary differential equations. The state space approach is in particular used for the basic problem in control theory, namely for the design of feedback control laws. The crux of the design problem is the variety of objectives which one wants to achieve at the same time.

(continued)

The lecture will be an attempt to look on the problem of modern feedback design from the mathematician's viewpoint. In addition some recent developments in nonlinear systems theory will be touched upon. These may provide new aspects for the design problem using models described by nonlinear differential equations.

I Knowles

Eigenvalue problems and the Riemann zeta function

In this lecture we consider some recent work relating to the problem of constructing an eigenvalue problem, the eigenvalues of which include the complex zeros of the Riemann zeta-function.

M A Kon

Superposition principles and pointwise evaluation of Sturm-Liouville eigenfunction expansions

Given a Sturm-Liouville eigenfunction expansion $f(x) \sim \sum_n a_n u_n(x)$ (either regular or singular), and $\phi(\lambda)$ such that $\phi(0) = 1$, the expansion is ϕ -summable in a given (pointwise or L^p) function topology if $\lim_{\epsilon \rightarrow 0} \sum_n a_n u_n(x) \phi(\epsilon n) = f(x)$.

This has been studied classically as well as in more recent integral (continuous spectrum) formulations. We illustrate some powerful applications of the superposition principle of summability, i.e., that if the expansion is ϕ_1 -summable and ϕ_2 -summable, then it is $\alpha_1 \phi_1 + \alpha_2 \phi_2$ -summable if $\alpha_1 + \alpha_2 = 1$. If ϕ is analytic and satisfies some minimal constraints, we use the representation of ϕ via Cauchy's theorem along with so-called resolvent summability to prove that the expansion of f is ϕ -summable in L^p ($1 \leq p < \infty$) and pointwise on the Lebesgue set of f . In order to consider $\phi \in C^n(\mathbb{R})$ we similarly use the Fourier and Mellin integral representations of ϕ . We remark on extensions to higher order operators.

A M Krall

Differential equation with orthogonal polynomial eigenfunctions

In this paper, we develop the eigenfunction expansion theory of a self adjoint operator generated by a symmetric sixth order differential equation $L_6(y) = \lambda y$. This differential equation has regular singular points at $x = \pm 1$ and we show that we are in the limit-5 case. This means that two boundary conditions are needed at ± 1 to ensure a well-posed boundary value problem. Not many examples are known of such higher order singular differential equations. The example that we give is interesting because the eigenvalue problem $L_6(y) = \lambda y$ has a sequence of polynomial solutions that are orthogonal on $[-1, 1]$ with respect to the weight distribution

$$w(x) = \frac{1}{A} \delta(x+1) + \frac{1}{B} \delta(x-1) + C.$$

K Kreith

Qualitative theory for hyperbolic partial differential equations

Motivated by the well developed theories of oscillation and asymptotic behaviour for $u'' + f(x,u) = 0$, corresponding questions will be considered for characteristic initial value problems associated with $u_{xy} + f(x,y,u) = 0$. Several known oscillation criteria for the existence of solutions which remain bounded for large values of x and y .

M K Kwong

Second order linear and nonlinear oscillation results

We are interested in conditions that guarantee that all solutions of the second order equation: $x''(t) + q(t)x(t) = 0$, or more generally of the nonlinear Emden-Fowler type equation: $x''(t) + q(t)|x(t)|^{\gamma} \operatorname{sgn} x(t) = 0$, have an infinite number of zeros on the half line $t \geq 0$.

The linear theory is more complete. We look at $Q(t)$, an indefinite integral of $q(t)$. The concept of asymptotic constancy (nonconstancy) is introduced. The main result is:

THEOREM Under a further mild bounded-belowness condition on Q , if Q is asymptotically nonconstant, then the solutions of the linear second order equation are oscillatory. If however Q is asymptotically constant, most of the classical oscillation techniques can be extended with minor modifications.

Similar approach in the nonlinear theory yields results much better than existing ones.

Another simple but extremely useful new result in the linear theory is a telescoping principle. Roughly speaking, by cutting off parts of $[0, \infty)$ in which Q is negative we only increase the oscillation.

The above are partly joint works with Professor A Zettl and partly joint works with Professor James S W Wong.

H Lechoslaw

On application of the method of the partition
of the unity for integral equations

Some initial-boundary value problems for a number of differential equations of physics, mechanics and technology are reduced to integral equations (e.g. potential's and Fourier's problems).

In this lecture a certain method based on the partition of the unity on compact manifold is proposed.

The considerations are introduced in the class C and L^p .

R T LewisA comparison theorem for quasi-accretive operators in a Hilbert space

Let $T: H \rightarrow H$ be a linear operator in the Hilbert space H with a domain $D(T)$ that is dense in H . If the numerical range, $\{(Tu, u): u \in D(T), \|u\| = 1\}$, of T is a subset of a half-plane $\{\xi \in \mathbb{C}: \operatorname{Re} \xi \geq \gamma\}$ for some real number γ , then T is said to be quasi-accretive. (If $\gamma = 0$, T is said to be accretive and $-T$ is dissipative). As a consequence, T is closable. We denote the closure of T by \tilde{T} .

In this lecture we first establish a theorem that allows us to locate the essential spectrum $\sigma_e(\tilde{T})$ of \tilde{T} by comparing $\operatorname{Re}(Tu, u)$ or $\operatorname{Im}(Tu, u)$ with quadratic forms associated with selfadjoint operators. Then the $\operatorname{Re} \lambda$ or $\operatorname{Im} \lambda$ of $\lambda \in \sigma_e(\tilde{T})$ can be compared with points in the essential spectrum of the selfadjoint operators.

We illustrate the result with certain differential operators.

S-O Londen
On the asymptotics of some Volterra equations with
locally finite measures and large perturbations

In this lecture we analyze the asymptotics of the scalar Volterra convolution equation

$$(1) \quad \frac{dx}{dt} + \int_{[0,t]} g(x(t-s)) d\mu(s) = f(t), \quad t \in \mathbb{R}^+ = [0, \infty), \quad x(0) = x_0.$$

In (1) g and f are given functions, μ is a given positive definite Borel measure on \mathbb{R}^+ and $x(t)$ is the unknown solution. In particular we concentrate on the case when both μ and f are large; that is μ is only locally finite and f vanishes at infinity but $f \in L^p(\mathbb{R}^+)$, for some $p < \infty$, does not necessarily hold. Our analysis depends on some results on the global size of the bounded solutions of certain limit equations

$$y(t) + \int_{\mathbb{R}^+} g(y(t-s)) a(s) ds = 0, \quad t \in \mathbb{R},$$

associated with (1).

A C McBrideIndex laws for some ordinary differential operators

A method is proposed for defining fractional powers of operators satisfying a relationship involving the Mellin transform. Index laws for these fractional powers are examined. As a special case, the theory is applied to a class of linear ordinary differential operators on $(0, \infty)$ and an indication given of a framework

for these differential operators are related to well-known results for the gamma function and the Gauss hypergeometric function, while an apparently new formula in the fractional calculus is unearthed en route.

P A McCoy

Converse initial value problems for a class of heat equations

In the classical initial value problem for the heat equation

$$\left\{ \partial_t - \partial_x [(1-x)^{\alpha+1} (1-x)^{\beta+1} \partial_x] \right\} \Psi(x,t) = 0, \quad -1 < x < +1, \quad t > 0$$

with parameters $\alpha \geq \beta$ and $\beta \geq -1/2$ or $\alpha + \beta > 0$, "arbitrary" initial data

$$\Psi(x,0) = f(x), \quad -1 \leq x \leq 1$$

is specified and one seeks the temperature function Ψ solving the heat equation for which $\Psi \rightarrow f$ as $t \rightarrow 0^+$. This paper considers the converse problem of identifying the initial data from the solution. It is shown that the solution exists uniquely in the space H^* of initial data that are hyperfunctions on $[-1, +1]$ by constructing an isomorphism between H^* and the space H of temperature functions. Thus, solutions of the heat equation are identified with their generalized boundary values. An essential feature characterizes the temperature as a series whose terms depend on a sequence of continuous functions satisfying a growth condition on $[-1, +1]$. The characterization combines function-theoretic and special function methods to extend a converse to the Dirichlet problem for harmonic functions in the disk.

J R McLaughlin

Higher order inverse eigenvalue problems

The problem to be discussed is as follows. Suppose a mathematical model for a vibrating system results in a selfadjoint eigenvalue problem of the form

$$w^{(4)} + (Aw^{(1)})^{(1)} + BW - \lambda w = 0$$

$$\sum_{i=1}^4 \alpha_{ij} w^{(i-1)}(0) = 0 = \sum_{i=1}^4 \beta_{ij} w^{(i-1)}(1), \quad j = 1, 2.$$

Suppose A, B and possibly α_{ij}, β_{ij} , $i = 1, 2, 3, 4$, $j = 1, 2$, are unknown but some spectral data can be obtained (possibly experimentally). Extensions of previous work by the author will be discussed, dealing with construction of the unknown coefficients from the spectral data and the dependence of the coefficients on the data.

A B Mingarelli

Indefinite Sturm-Liouville problems

The scalar boundary problem

$$-y'' + q(t)y = \lambda r(t)y \quad (1)$$

$$\begin{aligned} \cos \alpha y(a) - \sin \alpha y'(a) &= 0 \\ \cos \beta y(b) + \sin \beta y'(b) &= 0 \end{aligned} \quad (2)$$

where $0 \leq \alpha, \beta < \pi$; $q, r \in C(a, b)$ are real-valued is studied in the case when the coefficients of (1) give rise to the non-definite or "Indefinite case", that is, when (1-2) is neither "right" nor "left-definite" in current terminology.

Since, in this case, (1-2) may admit non-real eigenvalues, some results are derived regarding their number and some of the qualitative properties of the associated non-real eigenfunctions. Sufficient conditions for the non-existence of non-real eigenvalues are derived.

These results complement those contained in an old and somewhat forgotten paper of Ralph Richardson (Amer. J. Math. 40 (1918), 283-316) who pioneered the basic research of the indefinite case.

S E A Mohammed

The infinitesimal generator of a stochastic functional
differential equation

Let $r > 0$, $J = [-r, 0]$ and $C \equiv C(J, \mathbb{R}^n)$ be the Banach space of all continuous paths $\eta : J \rightarrow \mathbb{R}^n$ under the supremum norm. On a given probability space let $\{\eta_x(t) : t \geq -r\}$ be the solution of the stochastic FDE

$$\begin{aligned} d\eta_x(t) &= H(\eta_x)dt + G(\eta_x)dw(t) \quad t > 0 \\ \eta_{x_0} &= \eta \in C(J, \mathbb{R}^n) \end{aligned}$$

where $H : C \rightarrow \mathbb{R}^n$, $G : C \rightarrow L(\mathbb{R}^m, \mathbb{R}^n)$ are bounded Lipschitz maps, $\{w(t) : t \geq 0\}$ is m -dimensional Brownian motion, and $\eta_{x_t} \in C$ is defined almost surely by $\eta_{x_t}(s) = \eta_x(t+s)$, $s \in J$, $t \geq 0$. Then $\{\eta_{x_t} : t \geq 0\}$ is a continuous time-homogeneous C -valued strong Markov process. The space C_b of all bounded uniformly continuous functions $\phi : C \rightarrow \mathbb{R}$ is weakly topologized through the bilinear pairing $C_b \times M \rightarrow \mathbb{R}$, $(\phi, \mu) \rightarrow \int_C \phi(\eta) d\mu(\eta)$: where M is the B -space of all finite regular Borel measures on C given the total variation norm. Let $A : D(A) \subset C_b \rightarrow C_b$ be the weak infinitesimal generator of the semi-group $\{P_t\}_{t \geq 0}$ on C_b defined by

$P_t(\phi)(\eta) = E\phi(\eta_{x_t})$, $\phi \in C_b$, $\eta \in C$, $t \geq 0$. Augment the state space C by forming the complete direct sum $C \oplus F_n$, with $F_n = \{v\chi_{\{0\}} : v \in \mathbb{R}^n\}$, $\chi_{\{0\}} : J \rightarrow \mathbb{R}$ the characteristic function of $\{0\}$ and $\|\eta + v\chi_{\{0\}}\| = \|\eta\|_C + |v|$, $\eta \in C$, $v \in \mathbb{R}^n$. Define the canonical shift semi-group $\{S_t\}_{t \geq 0}$ on C be setting $S_t(\phi)(\eta) = \phi(\tilde{\eta}_t)$,

$t \geq 0$, $\phi \in C_b$, $\eta \in C$, $\tilde{\eta}(u) = \eta(0)$ for all $u \geq 0$, $\tilde{\eta}|_J = \eta$. Symbolize by $S : D(S) \subset C_b \rightarrow C_b$ the weak infinitesimal generator of $\{S_t\}_{t \geq 0}$; then we have

Theorem Let $\phi \in D(S)$ be C^2 with $D\phi$, $D^2\phi$ bounded and $D^2\phi$ Lipschitz on C . Then $\phi \in D(A)$. If $\{e_j\}_{j=1}^m$ is any basis for R^m , then

$$A(\phi)(\eta) = S(\phi)(\eta) + \overline{D\phi(\eta)} \{H(\eta)X_{\{0\}}\} + \frac{1}{2} \sum_{j=1}^m \overline{D^2\phi(\eta)} \{G(\eta)(e_j)X_{\{0\}}, G(\eta)(e_j)X_{\{0\}}\},$$

for all $\eta \in C$. $\overline{D\phi(\eta)}$ (and $\overline{D^2\phi(\eta)}$) are the weakly continuous linear (bilinear, resp.) extensions of $D\phi(\eta)$ ($D^2\phi(\eta)$) to $C \otimes F_n$.

P Morales

Generalization of the Hukuhara-Kneser property for some
Cauchy problems in Banach spaces

Let X be a not necessarily reflexive Banach space, let B be a closed ball in X with center x_0 , let $I = [0, a]$ be a compact interval of the real line, and let $f : I \times B \rightarrow X$. A recent result establishes that the Cauchy Problem (CP) $x' = f(t, x)$, $x(0) = x_0$ has a local solution if f is weakly continuous, strongly bounded and β -Lipschitzian, where β denotes the measure of weak noncompactness. In this lecture we present a delicate characterization of the solution set of (CP) refining the Hukuhara-Kneser property already known when X is reflexive. A discussion of further applications of our approach is also given.

M Nakao

On solutions to the initial-boundary problem for perturbed
porous medium equation

The existence, nonexistence and asymptotic behaviour of global solutions are discussed for the initial-boundary value problem:

$$\frac{\partial}{\partial t} u - \Delta u^{p+1} - u^{\alpha+1} = 0 \quad \text{on } \Omega \times (0, T)$$

$$u(x, 0) = u_0(x) \quad (u_0 \geq 0), \quad u(x, t)|_{\partial\Omega} = 0 \quad \text{and } u \geq 0,$$

where Ω is a bounded domain in R^n , α and p are nonnegative numbers such that (i) $p > \alpha$ or (ii) $0 \leq p < \alpha \leq (p(n+2) + 4)/(n-2)$ ($n \geq 3$), $0 \leq p < \alpha < \infty$ ($n = 1, 2$).

F Neuman

A survey of global properties of linear differential
equations of the n-th order

A survey of basic facts from the theory of global properties of linear differential equations of the n-th order will be described. Algebraic, topological and geometrical tools, as well as the methods of the theory of dynamical systems and the theory of functional equations, make it possible to deal with global problems by contrast with the local investigations of Erioschi, Laguerre, Forsyth, Halphen, Stäckel, Lie, Wilczynski and others. E.g., the structure of global transformations of linear differential equations is described by algebraic means (theory of categories - Brandt and Ehresmann groupoids), global canonical equations are constructed using methods of differential geometry (including E. Cartan's method of moving frame).

The theory in question includes also effective methods for solving several special problems (often very visible without necessity of lengthy calculation, e.g., from the area of questions concerning distribution of zeros of solutions).

D Pascali

Strongly nonlinear evolution variational inequalities

Let $Q =]0, T[\times \Omega$ be the cylinder corresponding to a bounded domain Ω in \mathbb{R}^N and a fixed $T > 0$. We consider a bounded coercive operator A_t of pseudo-monotone type with respect to two Banach spaces (Y, X) and a lower-order perturbation $g(t, x, u)$, on which no growth restriction is imposed. For an element $f \in X^*$ and a closed convex set K of X , an existence result of a solution to the variational inequality.

$$\left(\frac{\partial u}{\partial t} + A_t u - f, u - v \right) + \int_Q g(t, x, u)(u - v) \geq 0 \quad \forall v \in K \cap L^\infty(Q)$$

is established.

D Race

On the essential spectrum of some linear 2n th order
differential expressions

The differential expression

$$\ell(y) = (-1)^n (p_0 y^{(n)})^{(n)} + (-1)^{n-1} (p_1 y^{(n-1)})^{(n-1)} + \dots - (p_{n-1} y^{(1)})^{(1)} + p_n y$$

is considered for $y \in L^2(a, \infty)$, $a > -\infty$. The coefficients are permitted to be complex-valued, making ℓ formally J-selfadjoint. The associated minimal operator T_0 is J-symmetric and has J-selfadjoint extensions determined by ℓ .

T_0 and its extensions all have the same essential (or continuous) spectrum. Under certain assumptions on the coefficients, such as

$$\int_J \operatorname{Re} p_k \geq -D_k \quad \text{for some } D_k \geq 0 \quad \text{and for all intervals } J \text{ of length } \leq 1$$

for $1 \leq k \leq n$, results will be given, which confine the essential spectrum to a half-plane, quadrant or half-line in the complex plane, depending on the stringency of the conditions placed on $\operatorname{Im} p_k$, $1 \leq k \leq n$.

These results also yield conditions under which the regularity field of T_0 is non-empty, a requirement which is essential in the development of the operator theory associated with \mathcal{L} .

A. G. Ramm

Basisness property and asymptotics of spectrum of some nonselfadjoint differential and pseudo-differential operators

1. A linear operator A with a discrete spectrum on a Hilbert space H has the basisness property if its root system forms a basis of H . We write $A \in R_b(H)$ if the basis is a Riesz basis with brackets (see e.g. [1] for definitions).

Basisness property will be discussed for i) the operators of the form $A = L + M$ on $H = L^2(D)$, where $D \subset \mathbb{R}^d$ is a bounded domain with a smooth boundary Γ , L is a selfadjoint elliptic operator of order ℓ , M is a (nonselfadjoint) differential operator of order $m < \ell$. It will be proved that $A \in R_b$ if $\ell - m \geq d$ (*). For ordinary differential operators $d = 1$ and (*) holds for any M such that $m < \ell$; ii) some pseudo-differential operators (e.g.

$Af = \int_{\Gamma} \frac{\exp(ik|x-y|)}{4\pi|x-y|} fdy$, $A: H \rightarrow H$, $H = L^2(\Gamma)$ and iii) for some abstract operators (e.g. $A = Q(I + S)$, where $Q > 0$ is compact, $\lambda_n(Q) \sim cn^{-r}$, $r > 0$, S is compact, $|Sf| \leq c|Q^b f|$, $rb \geq 1$).

2. Suppose that A is a linear operator with a discrete spectrum on H , and $B = A + T$. When does B have a discrete spectrum such that $\lambda_n(B) \cdot \lambda_n^{-1}(A) \rightarrow 1$ as $n \rightarrow \infty$ or $s_n(B)s_n^{-1}(A) \rightarrow 1$ as $n \rightarrow \infty$? This and similar questions will be discussed. In particular the remainder $\varepsilon_n \rightarrow 0$ will be estimated.

3. Some open problems will be formulated.

Reference

- [1] A. G. Ramm, Theory and applications of some new classes of integral equations, Springer, N.Y. 1980.

R Rautmann

On error estimates for non-stationary Navier-Stokes approximations

Semigroup methods lead to new error estimates completing the estimates which have been proved in [1, 2] by energy methods.

1. Rautmann, R Eine fehlerschranke für Galerkinapproximationen lokaler Navier-Stokes-Lösungen, ISNM 48, Birkhäuser Basel 1979, p 110-125.
2. ——— On the convergence rate of non-stationary Navier-Stokes approximations, Springer Lecture Notes in Mathematics 771, 1980, p 425-449.

T T Read

Sectorial second order differential operators

We characterize the property of being bounded below for operators defined by symmetric expressions of the form

$$-(py' + \bar{r}y)' - ry' + qy$$

on the half line $[0, \infty)$, and investigate various properties of sectorial operators for which this is the real part. Results include a limit-point criterion, a generalized form of Dirichlet index, an explicit characterization of the domain of the "Friedrichs extension" of operators defined by the differential expression on compactly supported C^∞ functions, and estimates of the numerical range of such operators.

H Røh

Spectral mapping theorems for dissipative C_0 -semigroup generators

Let $G : D(G) \subset H \rightarrow H$ be a densely defined maximal dissipative operator with compact resolvent on a complex separable Hilbert space H and $T(t) = \exp(Gt)$ the C_0 -semigroup generated by G . We prove that the spectral mapping theorem $\sigma(T(t)) = \exp(\sigma(G)t)$ holds if some eigenvalue conditions for G are satisfied and if either the set $\{x \in D(G) \mid \operatorname{Re}(Gx, x) = 0\}$ has finite codimension in $D(G)$ or if the resolvent of G is nuclear. The results are applied to the damped wave equation $u_{tt} + \gamma u_{tx} + u_{xxxx} + \beta u_{xx} = 0$, $t \geq 0$, $\beta, \gamma \geq 0$, $0 < x < 1$, with boundary conditions $u(0, t) = u_x(0, t) = u_{xx}(1, t) = u_{xxx}(1, t) = 0$.

B P Rynne

Bloch waves and multiparameter spectral theory

In a one parameter setting Bloch waves are generalized eigenfunctions of the periodic Schrödinger operator and are used to investigate the spectral properties of this operator. The generalization of the theory to a multiparameter setting is

D A Sánchez

Some preliminary results on periodic solutions of
matrix Riccati equations

The matrix Riccati differential equation is the matrix analogue of the scalar Riccati differential equation and occurs frequently in applications such as filtering theory, optimal control etc. While such topics as existence of periodic solutions, number of them, and possible periods have been extensively discussed for the scalar equation, virtually nothing is known for the matrix equation. Some preliminary results for the autonomous, homogeneous matrix Riccati differential equation are given along with some examples, and further directions of investigation are discussed.

R Saxton

Solitary and travelling waves in a rod

We consider the existence and analysis of travelling waves in a model equation for longitudinal motion in a rod of general nonlinear stress constitutive law. An application is found for growth conditions in incompressible elasticity.

P W Schaefer

Comparison principles for some fourth order elliptic problems

Global type ($u \leq v$ throughout some domain Ω) comparison theorems are developed for some fourth order elliptic differential inequalities. The theorems are consequences of maximum principles which are developed for suitable functionals on the solutions of the differential inequalities. These results are useful in approximating the solution to some fourth order elliptic boundary value problem.

K Seitz

Investigation in the theory of partial differential
equations of infinite order

Consider the following partial differential equations of infinite order

$$\sum_{n=0}^{\infty} \sum_{k=0}^n a_{k,n-k}(z,w) \frac{\partial^n F(z,w)}{\partial z^k \partial w^{n-k}} = 0 \quad (1)$$

where $a_{k,l}(z,w)$, ($k, l = 0, 1, 2, \dots$) are analytical in $|z| \leq R$, $|w| \leq R$, where R is a positive constant. Let $\phi(\hat{z}, \hat{w})$ and $\psi(\hat{z}, \hat{w})$ be analytical functions in $|\hat{z}| \leq R$, $|\hat{w}| \leq R$, where R is a positive constant.

.../continued

In this lecture we shall investigate the structure of the $z = \phi(\hat{z}, \hat{w})$, $w = \psi(\hat{z}, \hat{w})$ transformations which map (1), to

$$\sum_{n=0}^{\infty} \sum_{k=0}^n a_{k,n-k}(\hat{z}, \hat{w}) \frac{\partial^n \hat{F}(\hat{z}, \hat{w})}{\partial \hat{z}^k \partial \hat{w}^{n-k}} = 0 \quad (2)$$

where $\hat{F}(\hat{z}, \hat{w}) = F(\phi(\hat{z}, \hat{w}), \psi(\hat{z}, \hat{w}))$.

J K Shaw

Well-posed boundary problems for Hamiltonian systems of
limit point or limit circle type

A Hamiltonian system $J\vec{y}' = [A(x) + B(x)]\vec{y}$, $a < x < b$, singular at both endpoints a and b , is considered, where $A(x)$ and $B(x)$ are locally integrable Hermitian $2n \times 2n$ matrix functions on (a, b) $A(x) \geq 0$ and $J = \begin{bmatrix} 0 & -I_n \\ I_n & 0 \end{bmatrix}$, I_n being the $n \times n$ identity. The system is assumed to be of either limit point or limit circle type at the endpoints. That is, the number of linearly independent vector solutions \vec{y} which are of integrable square with respect to $A(x)$ towards a given endpoint is either n or $2n$. For \vec{y} to be of integrable square towards b , relative to $A(x)$, means that $\int_c^b \vec{y}^*(x)A(x)\vec{y}(x)dx < \infty$, where $a < c < b$, and similarly for the endpoint $x = a$.

This paper develops a theory of boundary problems for the two singular endpoint systems given above. Explicit boundary conditions are given at the endpoints, resolvent operators are constructed and unique solutions are given for the problems, thus showing them to be well-posed. The results given extend to Hamiltonian systems a theory of singular boundary problems for second order scalar equations due to K Kodaira. The theory given in this paper also encompasses the classical λ -dependent Wronskian boundary conditions of Titchmarsh.

B D Sleeman

An abstract multiparameter spectral theory

This lecture reports on joint work with P A Binding and A Källström.

We consider the eigenvalue problem

$$A_i x_i = \sum_{j=1}^k \lambda_j B_{ij} x_j, \quad 0 \neq x_i \in H_i, \quad i = 1 \dots k,$$

for self-adjoint operators A_i and B_{ij} on separable Hilbert spaces H_i . It is assumed that A_i^{-1} and B_{ij} are bounded with $B_{ij}A_i^{-1}$ compact. Various properties of the eigentuples λ_j and x_i are deduced under a "definiteness condition" weaker than those used by previous authors, at least in infinite dimensions. In particular, a Parseval relation and eigenvector expansion are derived in a suitably constructed tensor product space.

R A Smith

Poincaré's index theorem and Bendixson's negative criterion
for certain differential equations of higher dimension

Poincaré's theorem on the sum of the indices of a plane autonomous differential equation at its critical points inside a periodic orbit is here extended to the periodic orbits of higher-dimensional equations under certain conditions. These conditions are essentially the same as those used earlier to extend to higher dimensions the Poincaré-Bendixson theorem on the existence of periodic orbits. Under the same conditions, higher-dimensional extensions are also obtained of a theorem of Bendixson which excludes periodic orbits from regions in which the differential equation has positive divergence.

E Stephan

Solution procedures for three-dimensional eddy current problems

My lecture is based on a joint work with R C MacCamy (Carnegie-Mellon University, Pittsburgh, USA) and considers the scattering of time harmonic electromagnetic fields by metallic obstacles. Two ideas are developed. The first is a boundary integral procedure for the eddy current problem. The second is an asymptotic procedure which applies for large conductivity and reflects the skin effect in metals. The key to both methods is the introduction of a new integral equation procedure for the boundary value problem corresponding to perfect conductors. The perfect conductor problem involves solving Maxwell's equations in the region exterior to the obstacle with tangential component of the electric field zero on the obstacle surface S . Whereas all known integral equation procedures for the perfect conductor problem lead to integral equations of second kind, our method leads to a system of first kind equations and gives a simple procedure for the calculation of the tangential component of the magnetic field on S . This enables us to formulate an integral equation procedure for the interface problem where different sets of Maxwell equations must be solved in the obstacle and outside while the tangential components of both electric and magnetic fields are continuous across S . The equations which appear in our integral equation method involve pseudo-differential operators on S . Precise existence and regularity results for these are obtained. The asymptotic procedure gives an approximate solution by solving a sequence of problems analogous to the one for perfect conductors.

Both procedures admit of numerical implementation techniques like finite elements.

K Soni

On uniform asymptotic expansion of a class of integral transforms

Asymptotic expansions for the integrals of the type

$$K(x,a) = \int_0^a k(xt) f(t) dt, \quad x \rightarrow \infty$$

which hold uniformly in a when either $0 \leq a \leq \delta$ or when $\delta \leq a < \infty$ for some $\delta > 0$, are obtained. It is assumed that f has an algebraic singularity at the origin and $f(t) t^{\lambda-1}$, $\lambda > 0$, is locally absolutely integrable $[0, \infty)$. It is well-known that in general the asymptotic expansion of $K(x,a)$ when $a \rightarrow 0+$ cannot be obtained directly from the corresponding expansion when a is bounded away from zero (see Erdélyi, SIAM J. Math. Anal., Vol. 5, 1974). A similar situation may arise as $a \rightarrow \infty$ (see Wong, Quart. Appl. Math., Vol. 38, 1980). Analytic continuation of the incomplete Mellin integral of k , namely,

$$M[k,a,s] = \int_0^a k(t) t^{s-1} dt$$

provides a unified approach to this problem and shows how the different expansions are related.

A Z-A M Tazali

Local existence theorems for ordinary differential equations
of fractional order

In this paper we prove two local existence theorems by using both the Picard's method and the Schauder fixed-point theorem for the following initial-value problem:

$$g^{(\alpha)}(x) = f(x, g(x)) \quad (\text{almost all } x \in [a, a+h]) \quad (A)$$

with

$$g^{(\alpha-1)}(a) = b \Gamma(\alpha), \quad 0 < \alpha < 1,$$

where $g^{(\alpha)}(a)$ denotes the derivative of order α of a real-valued function g , $\Gamma(\alpha)$ is the gamma function where $\alpha > 0$, b is a real number and under suitable conditions of the function f . We also prove an existence theorem of the maximum and the minimum solutions for the initial-value problem (A).

Tung Chin-Chu

The study of limit-cycles for Poincaré type equations

L TurynPerturbations of periodic boundary conditions

We consider perturbations of the problem (*) - $x'' + qx = \lambda rx$,
 $x(0) - x(1) = 0 = x'(0) - x'(1)$ both by changes of the boundary conditions
 and by addition of nonlinear terms. We assume that at $\lambda = \lambda_0$ there are two
 linearly independent solutions of the unperturbed problem (*) and that $r(\cdot)$
 is bounded away from zero.

When only the boundary conditions are perturbed either the Hill's
 discriminant or the method of Liapunov-Schmidt reduces the problem to
 $0 = \det((\lambda - \lambda_0)A - \epsilon H) + \text{higher-order terms}$, where A and H are real
 2×2 constant matrices. We analyse the existence of curves $(\lambda(\epsilon), \epsilon)$ of
 eigenvalues for this problem of linear perturbation and give as an example a heat
 problem with $H = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

The method of Liapunov-Schmidt is used to analyse the full nonlinear problem.
 Conditions are given for the bifurcation problem to be "generic" along the lines of
 Chow, Hale and Mallet-Paret.

A VanderbauwhedeDegenerate Hopf bifurcation for nearly-Hamiltonian systems

We consider autonomous, non-conservative perturbations of Hamiltonian systems.
 The unperturbed system is supposed to satisfy the conditions of the Liapunov center
 theorem, and consequently has a one-parameter family of periodic orbits. We
 describe sufficient conditions which ensure that under the perturbation not only the
 singular point survives, but also a small nontrivial periodic orbit bifurcates from
 this singular point, similar to the usual Hopf bifurcation. The approach is based
 on a non-linear Liapunov-Schmidt method which was introduced in an earlier paper.

R Villella-BressanFunctional equations of delay type in L^1 spaces

Consider the functional equation of delay type

$$(FE) \quad x(t) = F(x_t), \quad t \geq 0, \quad x_0 = \phi,$$

with initial data $\phi \in L^1(-r, 0; X)$, X a real Banach space. We associate with
 (FE) the operator in $L^1(-r, 0; X)$,

$$A\phi = -\phi^1, \quad D_A = \{\phi \in W^{1,1}(-r, 0; X), \quad \phi(0) = F(\phi)\}$$

and relate the semigroup generated by A in $L^1(-r, 0; X)$ to the semigroup
 generated in $C(-r, 0; X)$ by solutions with continuous initial data. We deduce
 information on existence, regularity and asymptotic behaviour of solutions of (FE).

J WalterOn the foundations of thermodynamics

It is the aim of this lecture to stimulate the discussion of the foundations of classical equilibrium thermodynamics by giving a three-dimensional "graph-theoretic" representation of a certain set of proofs of the Fundamental Theorem of this theory to be found in the literature. Furthermore a new variant of such a proof will be sketched centering around the notion of a vector field and procuring a new physical interpretation of Kelvin's notion of "circulation" in a special case.

W L WalterGeneralized Volterra prey-predator systems

(joint work with Prof R Redheffer)

The lecture treats the stability problem for systems of ordinary differential equations of the form

$$\dot{u}_i = N(u) f_i(u_i) \left(e_i + \sum_{j=1}^n p_{ij} g_j(u_j) \right), \quad u_i(0) > 0$$

($i = 1, \dots, n$) and of the corresponding parabolic systems

$$\frac{\partial u_i}{\partial t} = Lu_i + N(u) f_i(u_i) \left(e_i + \sum_{j=1}^n p_{ij} g_j(u_j) \right),$$

where L is an elliptic differential operator. The former system is generalization of the system

$$\dot{u}_i = u_i \left(e_i + \sum_{j=1}^n p_{ij} u_j \right), \quad u_i(0) > 0.$$

It is assumed that the matrix (p_{ij}) is stably admissible, i.e., that $(a_i p_{ij})$ is negative semi-definite for some $a_i > 0$ and that this property is preserved by certain perturbations. Using essential graph-theoretic methods, three classes of matrices are obtained, which correspond to the cases: (i) the stationary point is unique and asymptotically stable, (ii) every stationary point is stable, but not asymptotically stable, (iii) there exist periodic solutions.

A WoodOn some conjectures on the deficiency index for
symmetric differential operators

We outline recent results obtained jointly with R B Paris on asymptotic solutions of symmetric differential equations of form

$$(-1)^m y^{(2m)} + \sum_{r=0}^q (-1)^r (\alpha_r x^{2r} + \beta_r y^{(r)})(r) = \lambda y \quad \text{on } (0, \infty)$$

where $m \geq q > 0$, α_r ($r = 0, 1, \dots, q$) and β_r are non-negative real numbers and λ is a complex number. These results are used to comment on two recent conjectures by J B McLeod and by M S P Eastham and C G M Grudnicwicz. It is also hoped to show how such results can provide information about the asymptotic nature of the Titchmarsh-Weyl coefficient $m_{rs}(\lambda)$ in the fourth-order case.

S D WrayOn a criterion for discreteness and semi-boundedness of
a second-order differential operator

The operator is derived from the symmetric differential expression

$$M[f] = w^{-1} \{ -(pf')' + qf \} \quad \text{on } [a, \infty),$$

where p, q and w are real-valued coefficients on the interval $[a, \infty)$ of the real line. Conditions on p, q and w that enable one to associate with M a selfadjoint differential operator whose spectrum is discrete and bounded below are given. An identity involving the Dirichlet integral associated with M is also obtained. The theory makes use of a unitary change of independent variable.

E M E ZayedAn inverse eigenvalue problem for the Laplace operator

First of all, suppose the eigenvalues, $(\lambda_n)_{n=1}^{\infty}$, $\lim_{n \rightarrow \infty} \lambda_n = \infty$ are known exactly for the Sturm-Liouville problem

$$y''(x) + \lambda y(x) = 0, \quad 0 < x < a, \quad (1)$$

$$\left. \begin{aligned} y(0) \cos \alpha - y'(0) \sin \alpha &= 0, \quad \alpha \in [0, \pi), \\ y(a) \cos \beta - y'(a) \sin \beta &= 0, \quad \beta \in (0, \pi]. \end{aligned} \right\} \quad (2)$$

Determine the unknown length "a" of the vibrating string and the unknown angles α and β .

.../continued

Secondly, suppose the eigenvalues $(\lambda_n)_{n=1}^{\infty}$, $\lim_{n \rightarrow \infty} \lambda_n = \infty$ are known exactly for the τ -periodic problem comprising (1) and the boundary conditions -

$$\left. \begin{aligned} y(a) &= y(0) \exp(i\pi\tau), \\ y'(a) &= y'(0) \exp(i\pi\tau), \end{aligned} \right\} \quad (3)$$

where τ is a real parameter such that $-1 < \tau \leq 1$, $i = \sqrt{-1}$. Determine the period "a" and the parameter τ .

These problems have been attacked by a careful analysis of the asymptotic behaviour of the trace function $\Theta(t) = \sum_{n=1}^{\infty} \exp(-\lambda_n t)$ for small positive t .

UNIVERSITY OF DUNDEE

Conference on ordinary and partial differential equations

29 March to 2 April 1982

ADDITIONAL ABSTRACT

L Collatz

Inclusion theorems for singular and free boundary value problems

In this survey lecture we deal with applications of the theory of differential equations to the numerical methods of calculation of the solutions of ordinary (O.D.E.) and partial differential equations (P.D.E.). There are many different methods of computing solutions. The most frequently used methods are perhaps the methods of finite differences, finite elements and variational methods. These methods give numerical approximate values for the solution, but it is for these methods usually very difficult or impossible to give lower and upper bounds for the solutions (so called inclusions theorems) which one can guarantee, and with which one can see how many of the digits the computed has printed out are right. This guarantee can be given in many (not too complicated) examples by using fixed point theorems of the functional analysis, approximation and optimization-procedures, monotonicity properties of the solution and monotonicity theorems for iteration procedures. This will be illustrated by many examples of linear and nonlinear O.D.E. and P.D.E. and integral equations, the most examples coming from applications in sciences. But in many applications there occur singularities, as for instance reentering corners, singularities of the coefficients a.o.; it is important to take care of the type of singularity otherwise the numerical results would be unsatisfactory. Also for free boundary value problems it was possible in not too complicated problems to calculate inclusions for the free boundary one can guarantee. A new theory deals with vector valued operators of monotonic type which gives in simple cases inclusion theorems also for derivatives of solutions which may be in some cases even more important than the values of the solutions itself.

the complete direct sum $C \oplus F_n$, with $F_n = \{v\chi_{\{0\}} : v \in \mathbb{R}^n\}$, $\chi_{\{0\}}: J \rightarrow \mathbb{R}$ the characteristic function of $\{0\}$ and $\|\eta + v\chi_{\{0\}}\| = \|\eta\|_C + |v|$, $\eta \in C$. Define the canonical shift semi-group $(S_n)_{n \geq 0}$ on C by setting $S_n(\phi)(\eta) =$

END

DATE
FILMED

2-83

DTIC

tions
re

the
to
ms)
the
not

es
will
and
.
ing
are
not

ic
f
f